Changing Risk Over Time

As explained in Chapter 9, the most common assumption is to use a single, constant discount rate. But if the underlying risk is expected to change over time, the discount rate will change as well.[1]

For example, assume that the principal product of the subject business is patented and that the patent expires two years following the valuation date. One would expect that net cash flows in year three and thereafter will likely change (e.g., profit margins will likely decrease as a result of increased competition following expiration of the patent).

But we would expect that the riskiness of those expected net cash flows would also increase (e.g., expected volatility of the cash flows will likely increase as the result of increased competition).

The net cash flows of a business are not independent of each other. The net cash flows of year three, for example, build on the net cash flow and business operations in years one and two. Expenditures on advertising and sales calls to prospective customers in year one and two created demand for the business's goods and service in year three and thereafter. Capital expenditure in years one and two provide the capability of meeting customer demand in year three and thereafter. In year three, even though the patent has expired, much of that year's net cash flow is still dependent on expenditures made in prior years.

That is, NCF_2 is at least partially dependent upon NCF_1; NCF_3 is at least partially dependent upon NCF_2 and NCF_1, etc.

Because the net cash flows are dependent (or conditional), the discount factors for later years are dependent upon prior year discount factors. Formula A91 shows how the net cash flows should be discounted:

Formula A9–1

$$PV = \frac{NCF_1}{(1+k_1)} + \frac{NCF_2}{(1+k_1)^2} + \cdots + \frac{NCF_n}{(1+k_1)^n} + \frac{NCF_{n+1}}{(1+k_1)^n \times (1+k_2)} + \frac{NCF_{n+2}}{(1+k_1)^n \times (1+k_2)^2} + \cdots + \frac{NCF_{n+m}}{(1+k_1)^n \times (1+k_2)^m}$$

where:

k_1	=	Discount rate during period 1 through n
k_2	=	Discount rate during period *n+1* and thereafter

[1] The following material is adapted from Shannon P. Pratt and Roger J. Grabowski, *Cost of Capital: Applications and Examples* 5th ed. (John Wiley & Sons, 2014). Used with permission.

As an example, assume that k_1 = 12% and k_2 = 20%. Applying Formula A9–1 we get:

Formula A9–2

$$PV = \frac{\$100}{(1+0.12)} + \frac{\$120}{(1+0.12)^2} + \frac{\$140}{(1+0.12)^2 \times (1+0.20)} + \frac{\frac{\$140 \times (1+0.05)}{0.20-0.05}}{(1+0.12)^2 \times (1+0.20)}$$

$$= \frac{\$100}{1.12} + \frac{\$120}{1.2544} + \frac{\$140}{1.2544 \times 1.20} + \frac{\frac{\$147}{0.15}}{1.2544 \times 1.20}$$

$$= \$89.29 + \$95.66 + \$93.00 + \frac{\$980}{1.5053}$$

$$= \$89.29 + \$95.66 + \$93.00 + \$651.03$$

$$= \$928.98$$

Thus, the estimated value of this investment is $929.

The changes in risk may be due to changes in the business risk of the investment (as in our examples) or due to changes in the amount of debt used to finance the investment.

Three-Stage Model

In some valuations the terminal value may not be correctly valued using a constant growth model into perpetuity as was used in formula 9–6. For example, the analyst may have been provided only a limited number of years of discrete net cash flow projections. The growth rate in those projections may exceed the sustainable long-term rate of growth in net cash flows.

The analyst has two alternatives. The analyst can extend the discrete projections, decreasing the growth rate in net cash flows until net cash flows in the final discrete period are increasing at the same rate of growth as the sustainable long-term rate of growth in net cash flows (i.e., until the discrete net cash flows reach a steady state rate of increase). Or the analyst can apply a two-stage capitalization model instead of the single-stage constant growth model. The two-stage capitalization model allows the analyst to apply two different rates of expected growth in net cash flows beyond the discrete period.

The analysis therefore becomes a three-stage model (discrete period stage plus a two-stage terminal value).

The following eight steps summarize how to implement the three-stage model:

Step 1 Determine an appropriate number of periods or years for which discrete projections of net cash flows can be made.

Step 2 Estimate specific expected net cash flows for each of the discrete projection periods (periods *1* through n).

Step 3 Estimate a sustainable rate of growth, g_1, in the net cash flows from the end of the discrete projection period, n, for the number of years, m, until the sustainable long-term rate of growth begins (stage-two of the capitalization model).

Step 4 Estimate a sustainable long-term rate of growth, g_2, in net cash flows from the end of stage-two forward.

Step 5 Use the stage two and stage three portions (in {}) of Formula A93 to estimate the future value of the terminal value at the end of the discrete projection period (the present value of net cash flows for periods $n+1$, $n+2$… n+m, increasing at rate g_1, and for periods $n+m+1$, $n+m+2$, etc. increasing at rate g_2).

Step 6 Discount each of the discrete net cash flows back to their present value at the discount rate (cost of capital) for the number of periods until it is projected to be received.

Step 7 Discount the terminal value (estimated in step 5) back to a present value for the number of years in the discrete projection period (the same number of periods as in the last discrete net cash flow).

Step 8 Sum the value derived from steps 6 and 7.

These steps can be summarized by the next formula, which assumes that net cash flows are received at the end of each year:

Formula A9–3

$$PV = \frac{NCF_1}{(1+k)} + \frac{NCF_2}{(1+k)^2} + \cdots + \frac{NCF_n}{(1+k)^n} + \frac{1}{(1+k)^n}\left\{ \frac{NCF_n(1+g_1)}{(k-g_1)}\left[1-\left(\frac{1+g_1}{1+k}\right)^m\right] + \frac{\dfrac{NCF_n(1+g_1)^m(1+g_2)}{(k-g_2)}}{(1+k)^m} \right\}$$

where:

$NCF_1 \ldots NCF_n$	=	Net cash flow expected in each of the periods 1 through n, n being the last period of the discrete net cash flow projections
k	=	Discount rate (cost of capital)
g_1	=	Expected sustainable growth rate in net cash flow, starting with the last period of the discrete projections, n, as the base year for m years
g_2	=	Expected sustainable long-term growth rate in net cash flow, starting with the last period of the discrete projections as the base year having increased at the rate g_1 for m years.

We would typically expect that g_2 was less than g_1.

For simplicity in applying Formula A9–3, we will just use a three-year discrete projection period. Let us make four assumptions:

Expected net cash flows for years 1, 2, and 3 (n = 3) are \$100, \$120, and \$140, respectively.

For years 4 through 6 (m = 3), based on the business's performance and industry and overall economic expectations, 8% average growth in net cash flow, g_1, appears to be a reasonable estimate of sustainable growth.

Beyond year 6, based on the business's performance and industry and overall economic expectations, 5% average growth in net cash flow, g_2, appears to be a reasonable estimate of sustainable long-term growth.

The appropriate discount rate for this investment is estimated to be 12%.

Substituting numbers derived from these assumptions into Formula A9–3 produces:

Formula A9–4

$$PV = \frac{\$100}{(1+0.12)} + \frac{\$120}{(1+0.12)^2} + \frac{\$140}{(1+0.12)^3} +$$

$$\frac{1}{(1+0.12)^3}\left\{\frac{\$140\times(1+0.08)}{(0.12-0.08)}\left[1-\left(\frac{1+0.08}{1+0.12}\right)^3\right]+\frac{\dfrac{\$140\times(1+0.08)^3(1+0.05)}{(0.12-0.05)}}{(1+0.12)^3}\right\}$$

$$= \frac{\$100}{1.12} + \frac{\$120}{1.2544} + \frac{\$140}{1.4049} +$$

$$\frac{1}{1.4049}\times\left[\frac{\$151}{0.04}\left(1-0.9643^3\right)+\frac{\dfrac{\$176\times1.05}{0.07}}{1.4049}\right]$$

$$= \$89.30 + \$95.66 + \$99.65 + \frac{1}{1.4049}\times\left[\$3,775\times0.1033+\frac{\$2,640}{1.4049}\right]$$

$$= \$89.30 + \$95.66 + \$99.65 + \frac{\$2,269.10}{1.4049}$$

$$= \$89.30 + \$95.66 + \$99.65 + \$1,615.13$$

$$= \$1,899.74$$

The estimated present value of this investment is $1,900.

For a comprehensive discussion of discounting conventions, see Chapters 4 and 5 in Shannon P. Pratt and Roger J. Grabowski, *Cost of Capital: Applications and Examples,* 5th ed. (Hoboken, NJ: John Wiley & Sons, Inc., 2014).